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ABSTRACT

The purpose of the conference was to define the role of the School Mathematics Study Group (SMSG) in the preparation of programs and materials for gifted students. The background papers presented at the beginning of the conference and the recommendations made at the conference are presented in this report. Topics covered in the papers include a history of studies attempting to define gifted people, summaries of studies concerned with acceleration and enrichment for gifted students, activities for gifted students, and the problems of organizing special programs for gifted students. The conference gave the highest priority to recommending that SMSG develop supplementary materials. More specific recommendations included writing units of topics for investigation and open-ended research problems for students, developing expository booklets and possible correspondence courses for the mathematically gifted, encouraging and organizing local and regional symposia, organizing NSF summer and inservice institutes for training teachers to work with gifted students, investigating means of identifying gifted students other than by IQ scores, and extending some of the above activities downward into the elementary school. (JP)

SCHOOL MATHEMATICS STUDY GROUP

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A CONFERENCE ON MATHEMATICS FOR GIFTED STUDENTS

October, 1967

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PREFACE

This conference resulted from a request by the Advisory Board of the School Mathematics Study Group that programs and materials in mathematics for gifted students be studied and that recommendations be formulated as to the appropriate role of SMSG in the preparation of such programs and materials.

The usual SMSG procedure in the case of such a request is to assemble a group of individuals, representing all parts of the mathematical community, for a period of time long enough for a thorough discussion and the formulation of such recommendations.

An ad hoc committee (Roy Dubisch, John Harvey, William McNabb, Richard Pieters, William Slesnick, and Marie Wilcox) designed the agenda and selected the participants for such a conference, and the conference was held in Chicago on October 27 and 28, 1967.

The first part of the conference was devoted to the presentation of four background papers. There was vigorous discussion among the participants following each paper. The participants then divided into two groups, one concerned with classroom programs and materials and the other with extra-curricular activities for gifted students, for a continuation of these discussions.

A number of tentative recommendations arising from these discussions were presented in a plenary session on the second day of the conference for further discussion. A list of the recommendations approved by the conference participants will be found at the end of this report. It should be noted that some of these are addressed to the School Mathematics Study Group, but that others are addressed to other organizations.

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Characteristics of the Gifted

Frederick J. McDonald

The study of mankind has been essentially the study of man's characteristics. Mankind has recognized, throughout its long history, that individuals differ from each other in a wide range of characteristics. Aristotle distinguished between the rational capacities of man and his animal faculties, the former being associated with powers of his mind, the latter with the powers of his sensory apparatus. The Greek philosophers distinguished personality characteristics by postulating that man was composed of different combinations of basic humors. If one had more bile, he was likely to have a bilious temperament; more phelgm, a phlegmatic temperament.

The basic problem of the study of human characteristics is readily apparent in these early illustrations. First, what are the critical, or essential, or the uniformly differentiating traits of human beings? Second, how are they related? Practically, mankind has wanted to know, "Was the intelligent man also the good man?" and, philosophically, men have asked, "What is the relation between wisdom and goodness, between knowledge and action, between what a man believes and what he feels?" That such questions continue to intrigue us is apparent when we recall that people continue to ask such questions as, "Are the gifted, that is, the more intelligent of human beings, well-adjusted emotionally, physically healthy, and socially well-adapted?"

The problem of how to assess the characteristics of human beings lies at the heart of the problem of describing the characteristics of the gifted. It would be misleading simply to list the kinds of characteristics of gifted children which have appeared in studies reported in the literature without careful attention to the method by which such "facts" have been determined. Much of the confusion about what constitutes giftedness or talent arises be-

cause sufficient attention is not given to the problem of how a particular trait or characteristic has been assessed, and what faith we can put in the correlations found among such traits. The general principles relevant to the assessment of traits are relatively simple and straightforward. Knowing them and applying them in analyses of studies of gifted children will help assess the validity of such studies.

It is also fairly obvious that, despite the enormous amount of study of gifted children that has occurred in the past sixty-five years, many popular misconceptions still remain. There is presently a great lack of faith in the ability of IQ tests to identify such children. However, little consideration is being given to the implications of rejecting the method of mental measurement represented in an IQ score. Similarly, there are many claims about the relationship between creativity and intelligence measures which have been accepted as facts because they apparently cater to some people's beliefs.

The study of the characteristics of gifted children remains, at present, an open-ended problem for which there are a number of relevant questions remaining to be answered. No one can state unequivocally what these characteristics are, nor can the relationships between various characteristics be spelled out definitively. Rather, what can be done is to indicate the kinds of relationships which seem to exist among these characteristics, the characteristics themselves, and to describe the conditions under which such relationships are likely to be found.

The purpose of this paper is, therefore, two-fold; one, to describe the method of attack on the problem of identifying the characteristics of the gifted; and two, to describe the major kinds of relationships among characteristics and the salient characteristics of gifted children.

The Early History of Mental Measurement

Prior to the 1900's there was no systematic way of measuring mental

abilities. Such a practical limitation did not deter individuals from attempting to analyze the characteristics of individuals of unusual endowment. The basic method was that of retrospective analysis of the character of individuals of known attainments. Usually, the source of the data for this analysis was historical records of different kinds, such as statements individuals may have made about themselves in letters, comments about them from associates and family members, and biographies prepared from materials available on the life of the individual.

A representative study of this kind is reported by Yoder (Yoder, 1894). Yoder identified fifty outstanding individuals, men whose eminence and achievements were widely, if not universally, recognized in the Western world. He determined what sources of information about these individuals were the most authoritative. His next step was to cull these sources for specific information about each of these individuals to determine the frequency with which certain events appeared in their lives, and the extent to which certain characteristics were common among them. Some of the individuals whom he studied were Edison, Bismarck, Beecher, George Eliot, Darwin, Tennyson, Emerson, John Stuart Mill, Garabaldi, Scott, Froebel, and Napoleon.

To the reader of today, perhaps the most interesting aspect of this study is the kinds of information which he chose to assess. For example, one of the questions which Yoder considered important was the age of the parents when the gifted child was born. You may recall that there had been a popular mythology about gifted children being born relatively late in the lives of their parents, a belief that the gifted was a pre-menopausal but fortunate error. Yoder found that his eminent men were born at any time in the child-bearing period of the parents; the mothers ranged in age from 18 to 44, with a median age of 29.8.

Another interesting question, one which continues to be of some scien-

tific interest, is whether or not there was a relationship between birth order and giftedness. Yoder found that his eminent men tended to be in the elder half of the siblings in a family.

One of the items of information which has since been substantiated in other studies is that there is no evidence the gifted person was particularly weak or physically handicapped. A common popular belief which seems almost impossible to exterminate, is that the gifted person has a strong brain and a weak back, or that the gifted person's intense absorbtion in the things of the mind leads to a weakening of his physical capacities.

Some of the curiosities in Yoder's information are that these eminent men tended to be more "solitary" in their childhood play. Yoder's Victorianism creeps through in discussing this "fact," because he evaluates the solitariness of their play as a positive way of coping with their sexual impulses in adolescence, thus leaving their minds free. Another curiosity in his findings is reported as a fact disconfirming what was apparently a popular belief, that great men were educated largely by their mothers. According to Yoder, great men were frequently educated by a single individual, though not necessarily their mother.

The most interesting aspect of this study for our present purposes is that it reveals all of the difficulties in ex post facto induction. First, there is the problem of sampling. Clearly, the men that Yoder chose were eminent men, so that little dispute is justified about the criterion though there may be disagreement on the choices. Historically, some of them seem to be of less significance today than they were three-quarters of a century ago, but there is little doubt that either over the long run of history, or within the relatively short span of their own time these men were outstanding in some way. But, with that observation, one must also recognize that the sample is a limited sample, and the criteria for definitive inclusion are not at all

clear. This lack of clarity is particularly apparent when one considers the scientists who were included; the group does not appear to be representative of the eminent men of science.

Another obvious weakness of the selection method is that the criterion for selection, namely recognized eminence, is not based upon the same kinds of accomplishments, and, therefore, may not be representative of the same kinds of abilities or personality characteristics. To be eminent in science and to be eminent in literature are not, may not, and probably do not require the same set of human characteristics as necessary pre-conditions. Similarly, the conditions which stimulate the development of these respective achievements are probably different. One is tempted to assume a high degree of similarity among men of eminence by virtue of the kinds of questions that Yoder asked, rather than by any supporting factual data that these individuals were more alike than they were different. As we shall see, this idea persists today. Many people fail to recognize that aptitude as measured by high IQ scores does not necessarily mean that an individual will be gifted in every respect, or that he will be motivated to achieve in every field or area of human endeavor.

Another obvious weakness of the Yoder study is the ex post facto logic of his method. He obviously falls prey to all the weaknesses of the inductive method. He is also particularly liable to a biased interpretation of his data by looking at those facts relevant to those questions which he considers important.

It is trivial to point out that Yoder's methods are basically non-quantitative. It is also obvious that his information is as good as his sources. His data are second-hand observations at best, and the objectivity of the observer is moot.

You may wonder why I am spending any time at all in discussing the Yoder

study. I think it is important to recognize that Yoder's method in principle represents the approach to the analysis of the characteristics of the gifted which has characterized most popular thinking. I do not hesitate to say that many educators in their everyday thinking still use essentially this method for arriving at conclusions about gifted children. I find in talking to teachers and administrators about gifted children that their ideas seem to be relatively untouched by the scientific information which has been developed about giftedness and gifted children.

Although some of the historical prejudices have disappeared in their original form, they have acquired modern-day variants. For example, the hoary myth that genius is near insanity while largely dispelled among educated people, reappears in the form of an assumption that gifted children are likely to have emotional problems. Given this assumption, it is relatively easy to rationalize this assumption as if it were a fact by arguing logically that gifted children, being different, are subject to a wide variety of frustrating circumstances. It is but a short step from this assumed fact to the conclusion that these children must suffer considerable emotional pain, which obviously leads to emotional maladjustments.

Much information shared by teachers and administrators about gifted children is largely reportorial in character. This information arises from the selected observations of teachers about a selected group of individuals; it is filtered through the the teacher's own perceptual system, modified by his biases, and reported as fact to his colleagues. It is appalling, when one is talking with teachers about gifted children in their class, to find that these teachers use almost exclusively their immediate and direct observations of the child as the basis for all of their judgments about him.

It serves no useful purpose to rant about this state of affairs. I call it to your attention because it appears to me to be widespread. Almost any

attempt to begin a program for gifted children requires a re-education of the adults involved about the nature of giftedness and about the characteristics of gifted children. I would expect almost any innovative program for these children to flounder at some point simply because the children do not fit the teachers' sterotypes of giftedness. For example, I observed a fourth grade teacher who expected a group of gifted children to be arithmatic whizzes by the time she began teaching them in the fourth grade. The fact that they made errors in calculation or were sloppy in their work came to her as a great surprise. Her solution to this terrible state of affairs was to resort to intensive drill on the fundamentals of arithmetic, a solution guaranteed to heighten the boredom of the children who had long since learned the basic concepts. Obviously, I am not ridiculing the importance of accuracy in arithmetical work.

I am simply pointing out that the teacher's solution to this problem was based upon her misconception of what a gifted child was likely to do. Interestingly enough, the children in the gifted program to which I am referring regularly do rather poorly on the school district's tests of arithmetic fundamentals. The fallacies inherent in some forms of induction are apparent if one were now to draw the conclusion that gifted children generally do poorly in arithmetic.

The most extensive studies of the kind that I have been describing were conducted by Sir Francis Galton in two volumes, and appeared in Hereditary Genius (Galton, 1869) and English Men of Science (Galton, 1874). Galton's contribution to the study of talent or genius is one of the most significant even though he was limited by not having quantitative tools. Galton's contribution was a new conception of what constituted genius. Prior to Galton's time, and in much popular thinking today, talent or genius is treated as a qualitative difference between individuals. That is, the gifted person is thought of as being in a special category, as somehow uniquely different from

ordinary individuals. Galton defined this uniqueness not in terms of qualitative differences among individuals but in terms of quantitative differences. The gifted individual was the person who had an unusual combination of personal traits which placed him at the extreme on a scale of differences.

Galton used Quetelet's probability notions to describe differences among individuals. He asked, "What is the probability that an individual will differ by a certain amount from the average?" (Since he had no quantitative methods for estimating such differences, his estimates of actual variation were purely speculative.) He began by grading individuals into categories "according to their natural gifts". Then using tables devised by Quetelet he estimated the number of individuals in these categories, achieving a distribution familiar to us as the normal distribution of ability. Few individuals fell in the superior categories; that is, genius was defined as the infrequent occurrence of a combination of traits. Thus, in one leap Galton introduced an entirely new conception of what constituted unusual ability.

It is important to note here that Galton's ideas were influenced by current conceptions of heredity. His study of Hereditary Genius indicated that men of eminence tended to come from the same families. This fact was for Galton sufficient evidence to argue for a hereditary basis to genius. Using the ideas of Darwin, he predicated a survival of individuals who passed on the necessary characteristics for genius, until a genius resulted who represented a superior combination of basic traits. These traits which comprised genius were intellect, zeal, and the power of work, characteristics which Galton had identified as salient in the lives of the men of eminence whom he had studied.

Before we return to the significant concept that Galton introduced into thinking about genius, it is worth noting here that the conception of genius as rooted in heredity continued to dominate thinking about talent all through

the first quarter of this century. The "nature-nurture" controversy in psychology dominated thinking about individual differences on through the 1920s. Early studies about giftedness were concerned with parcelling out the relative effects of heredity and environment, and data from these studies were considered relevant to the larger issue of the contribution of genetic factors and environmental factors to behavioral differences among individuals.

Many of the individuals interested in the study of the gifted, such as Terman, were avowed hereditarians to greater or lesser degree. They believed that talent had a substantial genetic base, though they may have differed among themselves as to the extent of the contribution of heredity. Many of them, like Galton, were supporters of the eugenics movement, believing that mankind ought to produce more and more superior individuals.

This hereditarian conception of genius has probably done more to defeat the development of programs for gifted children than any other single factor. Through the '20's and '30's, educational thinking was dominated by the conceptions of the progressive movement, a set of ideas which were shared by Americans generally and which were not confined exclusively to educators. The great American belief at this period was in the infinite perfectability of man. Moreover, a democratic society was committed to the continued improvement of the individual by manipulation of his social environment. It is incompatible with the belief that mankind can be improved to claim that some individuals will invariably be better than others and that some individuals are not likely to be dramatically improved by social manipulations. These ideas, compounded with fear of the rise of an elite, accounted for the periodic attacks on the notion of special talent and the idea of special programs for gifted children. So rational a man as Walter Lippman bitterly attacked Terman for his conceptions of the IQ and for this method of identifying individuals of special ability (Lippman, 1922).

The Development of Mental Measurements

Once Galton's ideas had been generally accepted, the next step was to develop methods for studying individual differences among people. Galton himself was intensively involved in anthropometric measurements. However, little that he did could be described as mental measurement. Largely, his methods were measures of sensory acuity and psychomotor performance.

The first real advance in mental measurement was made by Binet who, at the behest of the French government, was asked to identify children who were potentially educable and to distinguish them from those who needed special treatment because they were what we would now call retarded. Binet's contribution was twofold: first, he settled on a conception of what constituted mental ability, and second, he developed a method for identifying it. The following quotation describes Binet's conception of mental ability:

"It seems to us that in intelligence there is a fundamental faculty, the alteration or lack of which is of the utmost importance for practical life. This faculty is judgment, otherwise called good sense, practical sense, initiative, the faculty of adapting one's self to circumstances. To judge well, to comprehend well, to reason well, these are the essential activities of the intelligence . . . . indeed the rest of the intellectual faculties seem to be of little importance in comparison with judgment." (Binet and Simon, 1916, p. 42).

Two elements in this definition should be noted. First, the shift from psychomotor to judgmental processes is made. Clearly, Binet is saying that intelligence is not a psychomotor performance nor is it an immediate sensory response. Second, this factor is treated as a general factor, the absence of which leads to practical consequences such as the inability to manage one's life, and the presence of which leads to greater success in this

management. This definition determines the kinds of criteria against which mental procedures are validated. These have always been some measure of practical achievement, such as success in school.

Much of the controversy surrounding the use of IQ tests has revolved around these two ideas. There has been a long history of debate about whether or not intelligence is a general factor or a complex of specific factors. Also, the criterion against which most IQ tests have been validated has been success in school, leading many people to believe that IQ tests measure too narrow a range of factors.

However, if one looks at the development of mental measurements in their historical perspective, the idea developed by Binet is beautifully simple. His idea is that a mental ability test would measure a factor generally accepted as related to success in life. His next step was to develop tests which correlated with some criterion of success, such as the ability to profit from schooling. Individuals could be scaled according to their ability to profit from schooling, and, if their positions on this scale corresponded with their performance on tests of intellectual capacity, the latter could be used to identify those children most likely and least likely to succeed in school. This conception and the subsequent development of tests of mental ability is certainly one of the landmarks in the history of ideas and in the development of the scientific study of man.

Binet's tests and all subsequent developments have one characteristic in common. The tests have been age-graded; that is, tasks requiring mental ability have been administered to large numbers of children to determine at what age the majority of children pass these tests. Subsequently, in the process of developing the tests, such tasks are arranged by age levels. The measure of the child's mental ability, that is, his mental age, is determined by assessing how far he can proceed on this scale. When he reaches a point

at which he can no longer pass tests, he has reached the upper limit of his performance. The mechanics of scoring are not relevant here, but the mental age score represents an accumulation of months-units which are accumulated by passing tests at specific age levels. The resulting score is a mental age score.

The concept of mental age in itself would be of little use if we could not determine whether a particular mental age was superior or inferior at a given moment in the child's development. It was recognized from the beginning that chronological age was the base line of general development. Using Quetelet's notions, the vast majority of individuals should probably be average in mental development. Fewer will be above average, and fewer will be below average.

The simplest notion is to state that an average development as represented in his chronological age. That is, a child is average if his mental age corresponds to his chronological age; he is inferior if his mental age is below his chronological age; he is superior if his mental age is above his chronological age. The IQ score is simply an arithmetical transformation of this ratio achieved by multiplying the ratio by 100.

These ideas are relatively simple and may be familiar to many of you. I am reviewing them here to emphasize the essential characteristics of what is being measured when a child's IQ is assessed. If we are to understand what we mean by giftedness when giftedness is defined in terms of intelligence test scores, we must be precise on what the measurement of intelligence yields. An IQ score simply represents the degree to which the child's mental age exceeds, equals, or is less than his chronological age. His mental age is assessed by his performance on age-graded tasks. The assumption is that children of the same chronological age have comparable learning opportunities. Therefore, those who have learned more, represented by superior performance

on the mental ability tasks, presumably are more intelligent.

From the IQ measure itself, it is impossible to determine either what specific abilities these children may have, or what they may know in a particular subject. Obviously, a question of general interest is the extent to which intellectual development is correlated with other kinds of development, such as emotional development.

The history of the development of the intelligence test consists essentially in refining the ideas of Binet, in developing a variety of measures, and in studying the empirical correlates of mental test performance. Lewis M. Terman of Stanford University developed an American version of the Binet test, which is known as the Stanford-Binet. Three versions of this test have appeared and Terman's discussion of the problems of measuring mental ability are reported in his book, The Measurement of Intelligence (Terman, 1916). This test has gone through three versions, being successively revised each time, and re-standardized on the current generation of children.

It is the most widely used of the intelligence tests, the one against which almost all others are validated, and the one most commonly used in identifying gifted children. The reasons for its popularity in these respects is that the test has a long empirical history, that is, quite a bit is known about it as a measuring instrument. It is practical and easy to administer, though it requires a trained tester to administer it. It is usable across a wide age range, so that it is possible to identify young children who are very bright, as well as adolescents.

The Meaning of Giftedness

With these concepts in mind about the nature of mental measurement, it is now possible to describe what is meant by a gifted child. <u>A gifted child is a child who scores at least at the 130 IQ level or above on a test of</u>

mental ability. In probability terms, this means that the child is in the upper two percent of mental ability. From study to study and from program to program there is some variation in the kind of test used in identifying giftedness. Most programs will utilize multiple criteria to select gifted children. A child may be screened first by looking at his group intelligence test scores and assessing his teacher's estimates of his abilities. Then the child may be tested with an individual IQ test, such as the Stanford-Binet. Additional information is gathered on his reading and arithmetic achievement scores. Then, a judgment of his ability is made by evaluating the composite of data. In the State of California, the only criterion for being included in a gifted child program is scoring on the 130 IQ level, though in practice administrators check IQ scores against other measures of ability.

Thus it is clear what giftedness has come to mean. A gifted child is one who scores at a certain level on a test of mental ability. This fact is critical to remember in assessing the meaning of studies of giftedness and evaluations of programs for gifted children.

Studies of Characteristics of Gifted Children

Psychologists, having available a tool for mental measurement, began as early as 1921 an intensive study of children of special talent. Lewis M. Terman began his study of gifted children in that year, a study which has continued until today (Terman, 1925, 1947, 1959). Terman identified one thousand five hundred and twenty eight California children, ranging in IQ from 135 to 200, who were between the ages of 3 and 19 at the time of their identification. These children have been studied intensively over the past 45 years. They are now into mid-life, and the older among them have retired. An intensive record has been kept of their achievements, their life history, and such factors as their attitudes and values.

Terman's study answered a number of questions or put the lie to a number of claims about gifted children. As I mentioned earlier, it was long a popular belief that the gifted child was inferior physically. Terman's data indicates that his gifted children were not only adequate in this respect but were superior. They did not differ from normal groups or average groups in such factors as height and weight, but their physical well-being was superior, and they were notably free of physical and emotional liabilities.

These children displayed a wide variety of interests in their early childhood; they read more than the average child, covering a wider range of topics; they had more hobbies, were in more activities, and they were remarkably successful academically. Through the years they maintained this superiority. Among the men, 70 percent completed college, and the percentage receiving Ph.D. degrees was 5 times as great as a representative sample of college graduates. By and large, they entered occupations where their intellectual capacities were likely to be used, such as the professions and university life. They had more successful marriages, and were freer of emotional disturbances.

In the original studies and later work (McDonald, 1964), the most and least successful groups were compared. These comparisons suggest that at the extremes, the most successful differ markedly from the least successful (where success is defined as achieving eminence in an occupation requiring intelligence) on those characteristics which distinguished the group as a whole. Thus, the most successful are more successful in school, were in more activities, finished school earlier, had a higher level of personal and emotional adjustment, and were more achievement-oriented.

It would be a misinterpretation to assume that what Terman has discovered are the uniform or basic characteristics of gifted children. There

is a temptation to draw a composite picture of the "gifted child". What Terman has shown is that these children of high ability are likely to be "on the average" successful in a variety of ways. He has also put to rest such myths as those which claim that the intelligent child is likely to be emotionally maladapted, physically weak, withdrawn, and exclusively 'bookish'.

There are a number of limitations to the Terman study, successful as it has been in producing data about gifted children. First, the children are drawn from the middle and upper-middle classes in large part. There are few if any representatives of what we today call 'minority groups', with the exception of Jewish children. Second, the children were selected originally on the basis of teacher's recommendations. It is likely that teachers picked the child who more readily adapted to school life, and who probably was freer of obvious emotional and social disabilities. But if these limitations are kept in mind, it is clear that many of our assumptions about the effects of possessing high intellectual ability simply were false.

This does not mean that individual children will not be unhappy. There were individuals in the Terman group who were notably unsuccessful in many aspects of life, and who today are discontented and poorly adjusted individuals.

These observations lead to another problem which has been of considerable interest in the study of the gifted. That is, what characteristics of gifted children go together? Given the definition of giftedness as exceptional mental ability, as measured by tests what kinds of traits are likely to be associated with high intelligence? As interesting as this question is, there is no clear answer to it. There are obviously some traits which are uncorrelated with intelligence or only modestly correlated. Thus a bright individual may be highly social or may be anti-social.

The reason for this variation in general abilities is obvious. Emo-

tional stability depends only partially on intelligence. Social skill is only partially influenced by intellectual ability. It is most useful to think of intellectual ability as a mediating factor between the kinds of experiences to which a person is exposed and what he eventually learns. In general, the bright person is more likely to profit from experiences where intelligence is a critical factor in evaluating and profiting from this experience. He is no more or less likely to profit from an experience where intelligence is of little or of secondary help. For example, social skills may be acquired by being exposed to models of appropriate social behavior, by being reinforced for acquiring appropriate social skills. One would expect an intelligent person to be more perceptive or more acute in his observations of models, more aware of reinforcement contingencies. But his basic social skills are more likely to be acquired on the basis of his reinforcement history than on the basis of any native perspicacity in understanding social relations.

Creativity and Intelligence

In recent years, great interest has developed in the relation between creativity and intelligence largely through the publication of a book by Getzels and Jackson (Getzels and Jackson, 1962). These two investigators compared two groups of children, those who were high in intelligence as measured by intelligence test scores but relatively lower on measures of creativity, and a second group who scored higher on creativity tests but who were lower on intelligence tests. They found, surprisingly, that the two groups did not differ significantly in academic achievement. This comparability in school achievement and the associated discrepancy between creativity and IQ have lead many people to conclude that creativity and IQ are uncorrelated. Since creativity tends to be highly valued, many individuals feel that the use of IQ tests to identify the gifted necessarily leads

us to omit individuals of high creativity from gifted programs.

The Getzels and Jackson study has been criticized because sufficient attention had not been paid to the fact that they chose extreme groups and omitted overlapping groups. Secondly, such a conclusion is too strong for the data. The measures of creativity used were simple tests. These children had not demonstrated any unusual achievement of a creative nature, and there is not sufficient empirical evidence to demonstrate that the tests themselves predict unusual creativity.

An obvious conclusion seems to be that creative people will usually have a relatively high order of intelligence, but not all intelligent people will be creative.

Conclusion

Where does this discussion leave us in the analysis of the characteristics of the gifted? As has been pointed out repeatedly in this paper, two kinds of conclusions seem supportable; many of our beliefs about the gifted, that is many of the correlations that we assume among their characteristics, simply are not true; second, intelligence is probably relatively independent of other personality and special aptitude characteristics. Practically, this leaves us at this point -- when a program for gifted children is contemplated, one must decide what kinds of experiences are going to be provided, and what kinds of children with what kinds of aptitudes are likely to profit from these experiences.

Suppose that we envision a program for children who are mathematically or quantitatively gifted. The first step should be to determine the character of the experience that we are to provide -- which in this case would obviously be some type of special mathematical education. Our next question is, to what extent does the experience to be provided or learning to be achieved require special mathematical aptitude? It may be that simply

selecting children of a high order of intellectual ability will be sufficient to produce a group of children who are likely to profit from this particular program. Since mathematical aptitude tends to be positively and substantially correlated with general intellectual ability, selecting children of a high order of intellectual ability we will also pick up those children who have high mathematical aptitude. If we wish to be on the safe side, we can make some distinctions among those whose quantitative ability is known to be high, and separate out those whose general ability is high but whose quantitative ability is lower.

Having gone through some selection procedure of this character, what might we expect to find? In general, we would expect to find a group of children who can learn very quickly, who have a high order of conceptual ability, who can interrelate ideas, and who probably like to do so. However, we will not find that all of these children are interested in interrelating mathematical ideas, nor will all of them be interested in achieving a high order of mathematical excellence.

As a group, they should be eminently teachable, since they are likely to be relatively alert and well-adapted socially and emotionally. These, however, are general expectations, the exceptions are real and not altogether infrequent. If our selection process is buttressed by other information such as previous achievement in mathematics, or assessment of interest in mathematics, and assessments of emotional and social stability, our expectations are more likely to be confirmed.

It strikes me that it is far more important to recognize that if we pick children of a high order of intellectual ability they are, first of all, most likely to resemble children of their age and kind, and secondly, to differ widely on characteristics other than their intellectual abilities. A succinct statement on this point is given by Gallagher (Gallagher, 1960, p. 58). "There is a growing suspicion that the importance of intelligence

in the development of personality characteristics may have been overestimated. We have demolished the point of view that high intellect is associated with instability. In its place, however, we have added a concept that high intellect has actually aided a person in making a good adjustment. Now studies that rule out other factors, such as family stability or social status, seem to find less significant relationships in either direction between intellect and stability."

The gifted child is an individual. He is certain to have in common with other children labelled gifted only one characteristic, namely a high order of intelligence. Even the diversity of that intelligence becomes apparent when these children are studied individually. It is important to recall each time we label a child a "gifted child," we are describing where he is located on a scale of mental ability. We are not describing his personality. Many of our expectations about what his personality will be like are likely to be disconfirmed. His uniqueness as an individual is as real as that of any other person. His social uniqueness is found in his unusual high order of intelligence and his potential for success in using his ability.

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Comments on Acceleration and Enrichment for Gifted Students

Harry D. Ruderman

An important comparison of acceleration and enrichment is reported in "A Comparison of Mathematics Programs for Able Junior High School Students" by Miriam L. Goldberg, A. Harry Passow, David S. Camm, and Robert D. Neill, U. S. Office of Education, Bureau of Research, Project No. 5-0381 (1966).*

The bibliography contains 71 references. These references dealt with projects concerning methods teachers used to handle mathematically talented children. All of the devices used may be grouped either under "Enrichment" or under "Acceleration". The classification is, to some extent, rather arbitrary. Under enrichment we may list the following devices:

1. Clubs and seminars
2. Visiting lecturers, and specialists
3. Contests and Fairs
4. Projects and reports
5. Inserting selected topics for the selected class
6. Visits to computer centers and computer oriented topics
7. Publishing a school mathematics paper
8. Saturday and Summer Institutes
9. Differentiated Assignments
10. Selected readings, including the NCTM Yearbooks 27 and 28.
11. Submitting solutions to problems in the NCTM publication, The Mathematics Student Journal.
12. Quite often the class of mathematically talented children were simply given harder verbal problems and problems in computation.

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* Selected pages from this report are reproduced on the following pages.

Acceleration was achieved in a number of ways, some of which are listed below.

1. Moving algebra down into the eighth year.
2. Consolidating plane and solid geometry.
3. Consolidating eleventh year mathematics with advanced algebra.
4. Permitting children to take final examinations for the next term without having taken the course.
5. Permitting children to move ahead at their own rates in ungraded classes.
6. Consolidating grades 7, 8, 9 into two years.
7. Planning to move two years of college mathematics into the Junior and Senior High School with the Fehr Project. This is an effort directed to the top 2 percent and attempts to implement the Cambridge Report. This experiment is now in its second year.

Probably the greatest effort in evaluating Enrichment versus Acceleration was made by Goldberg-Passow project mentioned on the first page. This was a three year project directed to grades 7, 8, 9. Among its conclusions are the following:

1. Accelerated programs are better than enriched ones.
2. Contemporary programs are better than traditional ones.
3. Accelerated classes failed to demonstrate more positive attitudes towards mathematics than those in the enriched classes.
4. Academically able junior high school pupils achieved a higher degree of general mathematical competence and showed greater ability to cope with relatively unfamiliar material in contemporary-accelerated programs than in contemporary-enriched, standard-accelerated or standard-enriched programs.
5. The standard-enriched appeared to be least successful on both achieve-

ment accounts, but among the highest on the attitudes and self-rating measures. [Having been with the project as a consultant who tried to strengthen the teachers' backgrounds, the reasons for some of these conclusions could very well be that the teachers were not comfortable with the contemporary programs, especially when they were accelerated. This was their first experience with such a situation and their lack of confidence with both the material and pace may have contributed to the poorer reaction.]

A few additional comments may be in order regarding acceleration and enrichment.

1. For the most part teachers are familiar with what acceleration is but are not competent to judge what significant enrichment is.
2. Teachers need guidance in the selection of such materials judged as significant and welcome assistance.
3. There is a feeling that significant enrichment is acceleration, perhaps out of order.

Extracts From the Report "A Comparison of Mathematics Programs for Able Junior High School Students"

CHAPTER I

Background of the Study

Mathematics teachers and school administrators are perplexed by the choices available to them in the area of mathematics for the academically talented. The availability of a number of alternative programs, with no reliable information as to their value for the talented population, led to the design of a demonstration-research project to provide guidelines for content and procedural selection in junior high school mathematics.

This project emerged from a study conducted by the Talented Youth Project of the Horace Mann-Lincoln Institute of School Experimentation in cooperation with the Cheltenham Township (Pennsylvania) Public Schools. That study involved an assessment of the effects of varied instructional procedures and content on the mathematical achievement and attitudes towards mathematics of academically able junior high school students. In Fall, 1957, incoming seventh graders were selected for four comparable classes. Students were individually matched on intelligence, reading and arithmetic achievement, teacher assessment, chronological age and sex. For the four groups, the average I.Q. was 132-133; mean reading scores, 9.4-9.8; mean arithmetic achievement, 9.2-9.3; and teacher ratings, good-excellent. All four classes were pre-tested on a series of attitude and achievement measures.

During the first year, one of the Cheltenham classes was accelerated through a traditional arithmetic program, and, by the end of the year, demonstrated its readiness for the study of algebra by scores on standardized arithmetic tests and on a prognostic test in algebra. Two of the groups

followed the prescribed seventh grade material but spent time on a series of "enrichment" units covering the History of Numbers, Number Systems, Powers and Their Meaning. The fourth group served as a control. The following year, the Accelerated class completed the first year algebra and began the second algebra course. One of the Enrichment groups completed eighth-grade arithmetic and continued work on additional units dealing with Measurement and Statistics, Operation of Computers, Logic and Topology. The second enrichment group shifted to the University of Illinois Committee on School Mathematics (Illinois) program and completed Units I and II and began Unit III. The fourth group continued as a Control, following a standard eighth-grade mathematics program. In the ninth-grade, the Accelerated group completed the second year algebra course; the Illinois class completed Units III, IV and V. The Enriched class had a standard first-year algebra course with the addition of units titled Laws of Arithmetic, Logic, and Concepts of Inequality and Equations. Again, the Control class had the first-year algebra course commonly taught to students. The senior high programs were later modified to provide articulation with the junior high school experimental work.

At the end of the junior high three-year period (May, 1960), the STEP Mathematics Test, Form I-A, was administered to all four classes. The publisher's college freshman norms were used in assessing percentile ranks. In addition, a 24-item teacher made test, consisting of 6 items from each of the four approaches of programs was administered. On the STEP test, the Accelerated class scored significantly higher than the Enriched and Control classes (at or beyond the .05 level of confidence). Differences between the Accelerated and the Illinois classes on this test were not statistically significant. The Illinois class scored significantly higher than the Control class but did not do significantly better than the Enriched Class. The Enriched group's mean score appeared somewhat higher than that

of the Control class, but the difference was not statistically significant. On the teacher-made test, the Accelerated class scored significantly higher than did either the Enriched or the Control groups (beyond the .05 level of confidence). The Illinois group scored significantly higher than the Control class. Neither the difference between the Enriched and the Control classes, nor the differences between the Accelerated and the Illinois classes was significant.

The attitude inventory used at the beginning of the seventh grade was revised and readministered. The items on this inventory dealt with such topics as: Mathematics Impact on Society, Characteristics of the Mathematician, Mathematics as a Career, The Nature of Mathematics, Self-Appraisal of Mathematical Ability and The School's Effectiveness in Teaching Mathematics. The four groups differed significantly on the number of "positive" or "correct" responses given in some of the categories, but not in others. For most of the six categories, the order of the scores were similar to the pattern observed in the achievement test -- Accelerated and Illinois groups higher than the other two.

At the conclusion of the three-year Cheltenham study, the data indicated that acceleration and enrichment were not "opposing" concepts. On the contrary, acceleration, either through the standard curriculum or through newly developed curricula, seemed to provide talented students with meaningful and enriching experiences. Enrichment, on the other hand, seemed to become meaningful only when the students dealt with more advanced and more difficult concepts.

The Cheltenham Study involved only four classes with a single teacher for each of the programs. The findings raised many interesting questions which could not be answered due to the design restrictions. There were some differential outcomes in pupil achievement and attitude, but these might have been related to the mathematics program followed, to teacher

variables, or to other causes. The study demonstrated the need for doing something "extra" in mathematics for academically talented junior high school students, but it tended to be hypothesis-generating more than it did to provide clear directions for mathematics teachers and administrators. A partial replication which was initiated a year after the first one started, yielded essentially the same findings. A grant from the United States Office of Education Cooperative Research Program, together with continued support from the Horace Mann-Lincoln Institute of School Experimentation, made possible a demonstration-research undertaking to assess the relative effectiveness of varied approaches to the teaching of mathematics to academically talented students with a number of classes for each program.

Related Research and Review of the Literature

The outpouring of mathematics curriculum materials during the 1950's elicited two kinds of responses: one, "wait-and-see" and the other, "any-change-is-a-good-one." Some educators sought "convincing" evidence before making changes in their schools. Others made changes and looked for support for their choice. Both approaches indicate a need for studies regarding the appropriateness of suggested curriculum revisions (whether based on contemporary mathematical thought or rearrangement of traditional content), as comparisons between and among programs are few. This is especially true with respect to programs for the academically talented.

A thorough search of the literature dealing with mathematics education and the academically talented preceded the initiation of the Cheltenham Study. Much of the literature is exhortative without serious analysis of the factors involved in making adequate provisions for the talented. Some consists of surveys of programs and "promising practices." The body of research and experimentation is not extensive.

Surveys of Programs and Provisions

McWilliams and Brown (1957) described the provisions for mathematics education for superior junior high school pupils made in some 30 schools visited by the senior author. Class and out-of-class activities, special classes, acceleration, and resource materials were described as illustrative of provisions found. The findings from extensive surveys of provisions for teaching rapid learners in junior, senior and four-year high schools were reported by Jewett and Hull (1954) and by Frain (1956). The former surveyed public schools; the latter, Catholic schools. Multitrack programs and individualized instruction were described as the most widely used practices but no evaluation was made of the effectiveness of any of the administrative or instructional modifications included in either publication.

Bryan (1960) prepared a questionnaire to which 124 seventh and eighth grade teachers of mathematics responded. From an analysis of the responses and a study of the professional literature, Bryan suggested an accelerated mathematics program for gifted students which centered around concepts of number, symbolism, measurement and approximation, statistics and functions. She proposed their completing the first half of the ninth year by the end of the eighth grade. Roach (1958) studied the mathematics and science programs for gifted Indiana secondary school students and found that 95 percent of the 91 schools which responded to his questionnaire used enrichment as the chief method of providing for gifted students. Sixty-seven percent of the schools practiced homogeneous grouping in mathematics for the gifted.

Other surveys focusing specifically on mathematics programs for the gifted have been reported by Baumgartner (1953), Brinkmann (1954), and Gordon (1955). The National Council of Teachers of Mathematics (Cance, 1955) and the National Education Association's Project in Academically Talented Students (Hlavaty, 1959) both issued detailed reports on program provisions

for mathematics for the gifted. Both pamphlets contained descriptions of existing courses and proposals for improvement of programs but included no experimental findings. Blank (1964) reported a survey concerning content of advanced mathematics curricula.

Enrichment and Acceleration in Mathematics for Talented Junior High School Students

Curriculum developers suggest two learning "paces" for talented junior high school students -- acceleration or normal progress with enrichment. Usually enrichment is considered an addition to the normal program of studies, a broadening and deepening of learning experiences. Acceleration, on the other hand, connotes the movement of students through a program of studies at earlier years or in less time than average students take. In practice, enrichment in mathematics usually means additional problems, reports, or reading; while acceleration may mean algebra in the eighth grade or an advanced course at the senior class level. Both approaches are widely used with talented junior high school students.

A few studies have reported the results of experiments in which gifted students have been in enriched programs. Lessinger and Seagoe (1956) designed, tested and evaluated an enriched geometry program for gifted students. Six enrichment units were developed and taught to an experimental group of able youngsters in addition to the regular course. The same teacher taught the regular geometry course without the enrichment units to a control group. The experimental class showed a better grasp of the subject matter, acquired greater understanding of mathematics in general, were able to apply mathematics principles and insights better, showed more originality and creativity. However, the experimental group did not do better than the control in assimilating new mathematics materials.

An enrichment program in four classes of 98 selected students was

studied by Long (1958). In two classes, the talented pupils served as group leaders, gave special reports and projects, and presented new topics and materials. In all four classes, the same teacher taught the same topics and gave the same assignments and tests. In the two experimental classes which had the enriched program, both the talented and nontalented group surpassed the control groups in both achievement and attitude. Dorris (1963) used a specially planned program of traditional mathematics plus units from contemporary mathematics and found the program better suited for high ability groups than lower.

Elder (1957) and Devine (1960) described seminars as a means of enriching mathematics for gifted students at the junior and senior high schools. Alternative courses for a twelfth-grade mathematics program for able girls were developed and tested by Lawton (1960). A course in mathematical analysis seemed most desirable on criteria developed by Lawton who incorporated seminar work and individual projects into the program.

After two years of experience with seventh graders in central New York state schools Davis (1960) concluded that seventh graders seemed able to learn algebra. The results of an informal study were reported by Wells (1958) in which the achievement of capable students in an eighth-grade algebra class was compared with that of ninth-grade students taking a similar course. The able students achieved as well or better than the ninth-grade control class.

Culbertson (1961) studying an accelerated program in algebra, science reading and vocabulary, reported that groups covering a three-year program of studies in two years were as successful in algebra and reading but somewhat less successful in science and vocabulary as non-accelerated students. In general, achievement scores favored acceleration. Lang (1962) assessed pupil achievement and pupil, parent, teacher and administrator attitudes in

accelerated and nonaccelerated classes in a three-year study and found that all measures favored an accelerated mathematics program. However, a 40 percent attrition of students over the three years pointed up problems of initial identification and selection for accelerated programs. Strand (1962) studied the effects of supplemental instruction (15 minutes, twice per week for six weeks) in the form of units on sets, number bases, and comparison of addition in four different numeration systems. He found that the experimental group (26 eighth graders) compared favorably with the control class (15 eighth graders) who spent equal time on traditional mathematics.

In a study involving 66 eighth-graders and 62 ninth-graders enrolled in a beginning algebra course, Lawson (1961) found that the eighth-graders achieved significantly higher scores than did the ninth-grade pupils. All pupils were academically able. The classes were divided in two on the basis of I.Q., arithmetic achievement, and teacher recommendation. There were no significant differences in achievement gains between the upper and lower ability groups.

From a longitudinal study of the effects of acceleration and enrichment programs on attitudes of pupils in eighth grade mathematics and ninth grade algebra, Ray (1961) reported that the attitudes of accelerated students were more positive than those of students who had participated in enriched courses. Passow, Goldberg, and Link (1961) reported at the end of a three-year experimental program for gifted junior high school pupils, attitudes toward mathematics in general and toward the pupil's own mathematical ability increased more in the accelerated classes (whether traditional or contemporary) than in classes which followed a nonaccelerated traditional curriculum or even a program "enriched" by the addition of various units from contemporary mathematics.

Mathematics for the Talented Student

Writers sometimes cause confusion by speaking of mathematics programs for the academically talented student in the same terms as they do about programs for the mathematically talented student. "Academically talented" students include all those who will eventually specialize in the arts, sciences, business, the various professions, as well as in technology. "Mathematically talented" students are those academically talented students whose greatest proficiency lies in mathematics.

The age at which successful mathematicians become engaged in mathematics varies, but the majority seem to have made their choice early. Lloyd (1953) referring to a Swiss survey of 93 mathematicians, relates that all 93 had been committed to their life's work by the age of 26, all but four of them by the age of 18, and the vast majority before the age of 15, the age at which students leave American junior high schools. Little attention is given in the literature in formulating an operational definition of "mathematically talented." There appears to be a high, positive relationship between reading ability and success in mathematics courses. Such success is, of course, also related to IQ or general intellectual ability. Certain special qualities, such as those listed by Fehr (1954) -- high level abstract thinking, intellectual curiosity, persistent goal-directed behavior, virtuosity in mathematics often gained through individual study -- are often exhibited by successful mathematics students. While Guilford (1961) has identified specific components of intelligence which are essential for creative work in mathematics, these components apparently enter into creative efforts in other areas of knowledge as well. The identification of the potentially outstanding mathematics student is based on limited information.

Most programs in mathematics for talented students rely heavily upon

identification procedures based on intelligence, reading, mathematical aptitude, socio-economic status, teacher appraisal, and pupil interest. In his study of high school seniors, Jordan (1964) found that between 38.9% and 62.4% of the criterion variance could be explained by IQ and socio-economic status. Hegstrom (1963) reported that another 16% of the criterion variance may be accounted for by other variables used in selection such as teacher appraisal, past achievement, pupil interest and mathematical aptitude. Perhaps the restricted range of intelligence, the selection tests, and the evaluative criteria used by Hegstrom account for the small amount of variance he obtained. Fitzgerald (1963) concluded, after studying fifth, seventh, and ninth grade mathematics students, that "the ability of a child to learn mathematics is a unique characteristic of the child just as are height, reading skill, and chronological age." At the present time there is no simple measure or combination of measures which will allow wholly reliable prediction of mathematical ability.

In the absence of specific guides, what to teach academically talented students after identifying them is still a difficult decision. Johnson (1953) suggested that the most practical and the easiest thing for schools to do for academically talented students in mathematics is to make differentiated assignments. Assignment differentiation may involve additional study, research opportunities or accelerated coverage. Hartung (1953) points out that we have no evidence that what bright students are taught is "the best for them at their level of advancement, nor that other students of lower ability could succeed with the same sort of work."

There are many questions concerning the appropriateness of the various current mathematics programs for academically talented students. Klausmeier (1959) found that 1) retention of material learned is the same for low, high, and average ability groups if the mathematical tasks are put at the learner's achievement level; 2) the within-pupil variance in achievement is the same

for all ability groups; and 3) curriculum programs are typically oriented to average intellectual groups. Identification of talented students would enhance the efficiency of acquiring mathematical knowledge by "at least one grade level and possibly two for high IQ children by the end of the fifth grade."

A variety of practices designed to meet the needs of academically talented junior high school youngsters are found in the literature. Rudnick (1962) found that most provide for algebra in grade eight instead of grade nine, with analytics and calculus or statistics taking the place of former senior class offerings. Many studies show that algebra in the eighth grade is both possible and practical. Rosskopf (1958, 1961) does not agree with this type of provision for academically talented students, maintaining that an emphasis on mathematical structure, precision of language, work with concepts of equality and inequality, and the nature of proof are more appropriate learning experiences than traditional algebra.

Investigators have explored the possibility of using Joplin-type plans where ability groups, regardless of chronoligical age receive instruction together (Davis and Tracy, 1963); television instruction (Rollins et al, 1963); grouping procedures (K[illegible]eny, 1959; Cawelti, 1962); and self-instruction designed to provide enrichment, (Payne 1958). Either no evaluation or inconclusive evidence has been presented in testing the merits of the various suggestions.

Attitudinal changes have been investigated by Lyda and Morse (1963) and by Ellingson (1962). Both studies show that change in attitudes toward mathematics correlate with achievement and method of instruction. Ellingson reported that attitude scores were better predictors of performance in mathematics in high school as measured by the Iowa Tests of Educational Development than teacher judgment or initial scores from a similar battery of Iowa

Tests administered in the sixth grade.

In studies of various grouping patterns, i.e. homogeneously grouped *versus* heterogeneously grouped classes, Mahler (1961), Mulhern (1960) and Becker (1963) found no differences in mathematical achievement, but none of the investigators noted differentiation in subject-matter content offered students in the various grouping patterns. As in other studies, grouping pattern has little effect on the achievement of academically talented youth unless accompanied by differentiation in content or pace or materials.

Proposals of a more or less specific nature for improving mathematics programs for talented students have been advanced by Ahrendt (1953), Fehr (1959), Glennon (1957), Hartung (1953), Keaveny (1959), Lapino (1956), Lloyd (1953) and Rees (1953).

Specific Efforts to Provide for the Mathematically Talented: Local Programs, Summer Institutes and Seminars

Two additional types of provisions provide mathematically talented junior and senior high school youth with experiences beyond those found in the regular school program. One consists of extra classes outside or after school, Saturday or evening seminars. These are generally supported locally. The other consists of summer institutes held on college and university campuses, often encouraged and supported through funds from the National Science Foundation, private corporations, or foundations. In selecting students for such programs preference is usually given to those who are finishing the eleventh and twelfth grade. This criterion for selection stems in part from the fact that college personnel employed to teach the courses may be more comfortable with an age group akin to regular college students. The usual curricula offerings include set theory, analysis, symbolic logic, computer mathematics, and mathematical research. In both types of programs guest lecturers are used.

Relatively few institutions and seminars include junior high school students. Assumption Preparatory School, Worcester, Massachusetts (Van der Linden, 1962) and Rollins College, Winter Park, Florida (Wavell, 1962) are two schools which accept thirteen year old students. During the summers of 1962-4, Teachers College, Columbia University conducted a special summer program for highly gifted pupils who had completed the sixth grade. A portion of the program each summer was devoted to work in advanced mathematics.

Two programs open to talented junior high school students were found at Iowa Teachers College Laboratory School (Nielson, 1959) and at Illinois Normal State University (Flagg, 1961). The Iowa summer institute for bright ninth graders offered instruction in set theory, relations and functions, analysis of the plane, logarithms and slide rule, linear programming, probability and statistics. Illinois Normal made provisions through the academic year as well as in the summer months for bright junior high school students.

Most school programs emphasize acceleration of students into algebra at the eighth grade level, and this pattern remains the predominant one in curriculum design. When Baker (1962) surveyed the Michigan school systems to determine which kinds of provisions were being made for the mathematically talented youngsters of junior high school age, only 18% of the schools reported any special provisions at all. However, the 18% of schools which reported special programs enroll approximately one-third of the State's school population. Thus, at best, only about one-third of those who might be eligible have a chance to participate. Both enrichment and acceleration are practiced in the Michigan schools, with acceleration into algebra in grade eight the more common procedure.

Studies Involving Contemporary Mathematics Programs

Few studies have been reported which contrast contemporary with traditional programs. One study compared UICSM with SMSG; three studies con-

trasted achievement in UICSM classes with that made in traditional classes; a few have compared SMSG programs with traditional programs. Several SMSG studies were reported from evaluations at the Minnesota National Laboratory.

In a study of seventh and eighth grade students who attended SMSG classes for two years, Ziebarth (1963) found no difference between mean achievement of SMSG students and that of comparable students who followed traditional programs, as measured by the "Quantitative Thinking Test" of the Iowa Tests of Educational Development. However, significant differences in favor of the traditional program were obtained on the "Fundamental Operations Test" of the Iowa Every-Pupil Test of Basic Skills. Kraft (1962) evaluated the achievements of 92 classes, grades 9-12, using SMSG materials. On test-retest forms of STEP Mathematics the SMSG students did as well or better than did students nationwide.

No differences in student achievement were found by Shuff (1962) who compared pupils who had one year of SMSG with pupils who followed a traditional program. Using scores from STEP-Mathematics and COOP-Mathematics tests, he also reported finding no sex differences in achievement and no differences in pupil achievement attributable to teacher training, including attendance at summer institutes. In matched classes using SMSG materials, some of which had self-selection activities one or two days per week and others which had no such self-selection activities, Ebeid (1964) found no differences in achievement between the two groups although he did note improved attitudes in the experimental classes (self-selection activities) compared with those of the control classes.

In a study involving 623 pupils in grades five and eight comparing SMSG and traditional classes, Phelps (1963) found differences on the Dutton Attitude Scale. Fifth-grade SMSG pupils had better attitudes than their "traditional counterparts"; similar differences were not found at the eighth grade. SMSG program demands for rigor and precision of language apparently

did not have a negative effect on attitudes toward mathematics. Phelps also found a positive relationship between SMSG students' achievement scores and scores on measures of ability to think "creatively." In fact, he found that SMSG students at both grade levels scored significantly higher than traditional students on a Uses for Things Test (an instrument which calls for naming as many uses of two common objects as one can in three minutes). According to Phelps, students with higher IQ's tended to make higher scores on the "uses" or creativity sub-test.

In a comparison of SMSG and traditional classes from grade seven through ten, Williams and Shuff (1963) found that when intelligence was held constant, significant (.05 level) achievement differences on STEP tests favored the SMSG classes in the tenth grade only. For the eighth grade, scores tended to favor the traditional students.

Pate (1964) compared transactional patterns in SMSG and traditional classes. SMSG teachers used a higher proportion of divergent questions, spent more time elaborating on lessons, and had more interaction with pupils than did traditional teachers. Traditional teachers used more cognitive-memory operations. However, even though there was greater rigidity in the traditional classes, sufficient freedom existed to allow for pupil-pupil interaction.

Nelson (1962) studied the effects of varied textbook presentations on the mathematics achievement of high ability junior high school students (285 seventh and 460 ninth graders) in 14 schools. One experimental class of each pair used the SMSG R text (for college-capable) and the other used the SMSG M text (same topics but simplified for slower learners). He found that except for the very highest achievers, the M texts tended to facilitate learning of mathematics for all high-ability students.

In seeking evidence concerning SMSG student performance on Educational

Testing Services tests of traditional mathematical skills, Payette (1961) studied samples of seventh, ninth, tenth, eleventh and twelfth grade pupils both in SMSG and in traditional classes. On the basis of various analyses performed, he found that: 1) "students exposed to conventional mathematics have neither a pronounced nor a consistent advantage over students exposed to SMSG mathematics with respect to the learning of traditional mathematical skill;" 2) with respect to developed mathematical ability beyond that developed in traditional programs, "SMSG showed consistent extensions of developed mathematical ability;" and 3) that students at all levels of aptitude "can learn considerable segments of SMSG materials."

Rosenblum (1961) evaluating achievement in SMSG classes at the Minnesota National Laboratory, found that with ability level held constant, SMSG students did as well as other students. In seventh grade evaluations, SMSG pupils in seven of thirteen classes scored significantly higher on post-tests than their peers in traditional programs. Four other SMSG classes scored higher, but not significantly higher, than their control classes. The two control classes with higher means than their SMSG counterparts were not statistically different from the means of the two SMSG classes. However, differences in scores on retention tests between SMSG and "traditional" pupils were not significant, although SMSG mean class scores still remained higher. Comparisons done at the Minnesota National Laboratory in grades other than seventh grade were inconclusive, although SMSG student performance generally was higher than traditional student performance.

When the achievement scores of the top 20% of seventh grade students in SMSG and non-SMSG classes were compared by Mikkelson (1961) no differences were found between the groups in achievement as measured by both <u>STEP</u> and <u>California Arithmetic Reasoning</u> and <u>Arithmetic Fundamental</u> tests.

Loman (1961) studied the effectiveness of UICSM algebra and traditional algebra curricula with two middle-track ninth-grade classes of a three-track program. A statistically significant difference in favor of the UICSM group was obtained in the upper one-third ability level on the tests of understanding of basic mathematical concepts. No real differences were found at the middle or lower-third of intelligence. Nor were there any apparent differences in achievement of mathematical ability at any level of intelligence.

In comparing the achievement of approximately 1700 superior pupils in UICSM first year algebra classes with 700 pupils in "traditional" first year algebra classes, Tatsuoka and Easley (1963) found that pupils in both UICSM eighth- and ninth-grade classes performed significantly better on Cooperative Algebra Test (Elementary), Forms T, X, and Y. These tests measure traditional mathematical content. Since pupil aptitude was not the same for all groups in the study, an analysis of covariance was performed which equated all pupils' scholastic ability as measured by Differential Aptitude - Verbal Reasoning and Differential Aptitude - Numerical Ability. Both UICSM groups performed significantly better than non-UICSM pupils. When Tatsuoka and Easley compared eighth grade mean achievement with ninth grade means, they found eighth grade pupils did significantly better than ninth grade pupils, where both groups had studied UICSM materials. After removing the higher-scoring eighth grade sample, the ninth grade UICSM scores were still significantly higher than ninth grade traditional scores. The investigators concluded that UICSM material was adequate in preparing superior students to cope not only with UICSM tests but also with conventional tests.

In another UICSM investigation Tatsuoka and Comley (1964), using a matched-pairs design, compared the achievement of UICSM first year algebra students with non-UICSM first year algebra students in the Inglewood,

California schools. The _Cooperative Elementary Algebra Test_ and the _Cooperative Algebra I Test_ were used to assess "superior" pupil achievement in both eighth- and ninth-grade algebra classes in the study. Pupil-related variable considered in the covariate analysis of the two criterion scores were pupil assessments on _SCAT-Verbal_, _SCAT-Quantitative_, _California Algebra Aptitude Test_, _STEP-Mathematics_, and pupil sex. Teacher ratings made by a teacher's principal were also included in the analysis. Although UICSM student means were higher than those of the controls, the adjusted means which took into consideration all variables used in the analysis, were not significantly different. However, when the teacher rating score was excluded from the analysis, the UICSM means were significantly higher than the control group means. Tatsuoka and Comley suggested that the superior performance of UICSM pupils may be due to superior teachers.

In Conclusion

From the number of reports issuing from school systems, it is evident that more and more effort is being made to provide for able students in mathematics. The questions of what should be the nature of mathematics for the talented and what kind of special provisions should be made have not been adequately explored experimentally at any educational level. What research has been done is quite limited, often testing one modification against a traditional program for a brief time. The Cheltenham Study compared several approaches over a three-year period. However, only one teacher and one class followed each pattern. This present study field tested larger numbers of students and teachers with more varied approaches to the mathematics programs for talented junior high school pupils.

Purpose of the Study

The two purposes of this demonstration-research study were:

1. To assess differential outcomes of various approaches to teaching mathematics to academically talented junior high schools.

2. To develop guidelines for content and procedural selection in junior school mathematics.

Purpose of the Study

The purpose of the TYP Mathematics Study was to assess the relative effectiveness of varied approaches to the teaching of mathematics to academically talented junior high school pupils. The study was aimed at comparing the effects of standard, traditional mathematics programs with contemporary ones and of accelerated programs with enriched ones. Pupil achievement was defined in terms of (a) general ability to deal with quantitative relationships; (b) mastery of content of a particular mathematics program; and (c) ability to apply mathematics concepts and skills learned in one program to problems and processes derived from the content of other programs.

Hypotheses

The two hypotheses tested in this study were:

Hypothesis I -- Rapid sequential progress through a mathematics program is more effective than plans which provide either intermittent enrichment units (even when these are of an advanced nature) or depth study of normally paced sequential materials as measured by:

a. General mathematical competence;
b. Ability to apply knowledge to unfamiliar mathematical material;
c. Positive attitude toward mathematics.

Hypothesis II -- Compared with programs which follow a standard, traditional sequence, regardless of pace, programs which deal with contemporary mathematical content and methodology will result in:

a. Greater general mathematical competence;
b. More marked ability to apply knowledge to unfamiliar mathematical materials;

c. More positive attitudes toward mathematics.

Design of the Study

Population Selection

Pupils were selected on the basis of general intelligence (IQ above 120) and sixth grade reading and arithmetic achievement (scores approximately one and a half to two years accelerated). Attitudes toward mathematics, self-rating of ability, socio-economic status as well as interests, academic preferences, etc., were assessed but were not considered in pupil selection.

A total of 25 school systems, 51 classes and about 1500 pupils initially participated in the study. Complete, usable data at the end of the seventh grade were available for 1477 pupils. During the second year (eighth grade), 49 classes were involved with data available for 1271 pupils. By the end of the third year (ninth grade), due to normal attrition, changes in state requirements and overcrowded conditions in some schools, the number of classes dropped to 37 and the number of pupils, on whom all data were available for the three years, to 868.[a]

Program Selection.

In selecting programs for a comparative study, six were chosen which were presumably differentiated according to content (standard or contemporary) and teaching-learning pace (enriched or accelerated).

[a] Since only one of the Standard Accelerated classes followed a second-year algebra sequence in grade nine as originally agreed on, the 25 pupils of this class were not included in the sub-test or within program analyses.

Conclusions and Discussion

The results of the study only partially supported the two hypotheses. In most of the analyses of cross-program scores, the four accelerated programs exceeded the two enriched ones and the four contemporary programs exceeded the two standard, traditional ones. However, while the contemporary programs resulted in "greater gain in general mathematical competence" and in "the ability to apply knowledge to unfamiliar mathematical material", they failed to generate "more positive attitudes toward mathematics," in general, or to raise the pupils' assessment of their own mathematical ability above the level of the standard programs.

The accelerated programs generally exceeded the enriched ones on both mathematical competence and application of knowledge to new materials. However, these results were due to the higher scores of the three contemporary accelerated programs which outweighed the single standard accelerated one. Within the standard approach, the accelerated classes generally exceeded the enriched ones. Here, as for the hypothesis relating to the contemporary-standard comparisons, the accelerated classes failed to demonstrate more positive attitudes toward mathematics than those in the enriched programs.

In general, the study concluded that academically able junior high school pupils achieved a higher degree of general mathematical competence and showed greater ability to cope with relatively unfamiliar material in contemporary-accelerated programs than in contemporary-enriched, standard-accelerated or standard-enriched. Of all the program adaptations, the latter (standard-enriched) appeared to be the least successful on both achievement counts, but among the highest on the attitudes and self-rating measures.

Since over the three years, the three contemporary-accelerated programs proved about equally effective, it is not possible to compare the relative advantages of the two kinds of acceleration: beginning a sequence earlier than normal or working through a sequence more rapidly than normal. In both instances pupils are exposed to more varied and more advanced content than would otherwise be the case and are, thus, in a position to apply more extensive knowledge to the solution of unfamiliar problems. Nor can any conclusions be drawn regarding the relative merits of the SMSG and the UICSM programs when these are presented at an accelerated pace. In both programs the content and the methodology appear to have been more effective in fostering general mathematical ability and in enabling students to cope with relatively unfamiliar material than was true for the standard, traditional programs. Thus, contemporary-accelerated programs appeared to produce the best results, in terms of mathematical achievement, even though such programs apparently did not promote more positive attitudes toward mathematics.

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Extra-Curricular Activities in Mathematics

for

Mathematically Gifted Secondary School Students

Robert Kalin

Introduction

Extra-curricular activities in mathematics for the gifted fall rather naturally into four categories:

(1) Contests

(2) Group Study

(3) Independent Study

(4) Tutoring

A literature search, personal contact, and correspondence have been the principal means of gathering information for this report. The results are summarized in the following four sections in reportorial fashion, except for some occasional comments, summaries or suggestions.

Contests

There are many types of mathematical contests. Some are interscholastic, others intramural. They range in geographic scope from local to state to national to international. Another variation is in type of problem -- from a large sample of fairly easy multiple-choice questions administered in an hour on one extreme, to a small number of quite difficult problems administered on an open-book or take-home basis on another extreme.

In at least three states (Alabama, California, Wisconsin) mathematical competitions have been used as a means of searching out mathematical talent among high school students. At the University of Wisconsin, the Mathematics Department obtained National Science Foundation support in conducting an unusual contest in each of the past three years [4, pp. 412-16]. Four problem sets of five problems each were sent, on successive occasions, to various Wisconsin schools. A typical problem from the fourth set:

> If x and y are chosen to be integers, then $2x + 6y$ is divisible by 19 if and only if $5x - 4y$ is divisible by 19.

An attempt was made to select problems solvable without specific mathematical knowledge. This was a difficult task, as indicated by the occurrence of sixteen seniors and twenty juniors, as compared to only nine sophomores and one freshman in the top forty-six scorers who persevered through all four problem sets. These top students were invited for a one-day program on the University of Wisconsin campus, during which they attended a lecture, met the governor, and held some small group discussions with faculty members. In the third year, a project assistant was hired to supplement the program by lecturing to students and teachers throughout the state. Recently a former University of Wisconsin associate

developed an analogous program in Alabama through the auspices of Stillman College.

From 1946 through 1965, the Department of Mathematics at Stanford University sponsored a somewhat similar contest throughout the State of California. The first contest was held on a Saturday in April, 1946. The examination consisted of three extremely difficult problems taken from the subject matter of algebra, plane and solid geometry, and trigonometry, each requiring originality in its solution. Although the purpose was also to discover mathematical talent among high school students, administrative methods differed from those in Wisconsin in several respects [7, pp. 406-9]:

(i) Announcements of the contest were sent to principals of only large high schools of the state.

(ii) Only seniors were allowed to participate. The examination (three hours) was held in the school of each participant on a Saturday afternoon, with appropriate proctors on hand.

(iii) First prize consisted of a generous scholarship to Stanford. Honorable mention was given to other students presenting worthwhile attempts at solutions.

The Wisconsin and Stanford examinations owe much in their concept and their emphasis upon original problem-solving to the well-known Hungarian Eötvös competition. As reported in 1932 by Radó [25, pp. 85-90], this Hungarian contest differs in several basic respects:

(i) It is national in scope;

(ii) held in October, it is open only to secondary school graduates of the preceding June, (average age 18);

(iii) contestants are given four hours in which to solve these problems;

(iv) contestants may use any text they wish.

Professor Radó raises the question: "Is it possible to test by problems creative mathematical ability?" His (paraphrased) answers, pro and

con, are interesting:

(i) Psychological theories may be used to prove the impossibilities of such an attempt. E.g., there are truly able students who cannot solve anything within four hours in strange surroundings.

(ii) Some contestants have not only solved a problem, but have gone on to generalize the solution or improve the conditions of the question.

(iii) Several winners have become distinguished mathematicians. Others disappeared completely. But in this respect, it all seemed more than worthwhile.

(iv) The Eötvös prize is regarded as a real accomplishment by both professional mathematicians and the lay public.

Problems from the Hungarian competitions have been published in the Hungarian Contest Books, part of the School Mathematics Study Group New Mathematical Library.

In an interesting article describing many facets of mathematical education in the U.S.S.R. as of 1957, Gnedenko [10, pp. 384-403] reported briefly on the Olympiads held in his country each year. An Olympiad has three aspects: (1) group meetings held on university campuses for school students, (2) lectures for these students by university mathematicians, and (3) contests.[1] The competitions have these features of interest:

(i) They are held in several cities.

(ii) Each competition is divided into two rounds. Only those who do well in the first round participate in the second.

(iii) The problems are like those in the Stanford, Wisconsin, and Hungarian exams. (Gnedenko gave a goodly number of these in his article.)

(iv) In 1953, there were 1,350 students in Moscow taking part in round one, of whom 507 qualified for round two. Of the 262 successfully completing both rounds, 3 obtained first class, 15 second class, 24 third class and 69 got certificates of merit.

[1](Only the third part is reported on here; see the next section, <u>Group Study</u>, for information about the others.)

Of special interest is Gnedenko's report that there are newspapers for youngsters in which they publish mathematical articles and propose ingenious problems for other students.

The Hungarian and U.S.S.R. competitions are apparently related to the International Mathematical Olympiads for Students of European Communist Countries reported by Wirszup [39, pp. 303-16]. At least four of these international competitions have been held, the fourth in 1962. Some characteristics of these contests:

(i) Teams come from Bulgaria, Czechoslovakia, East Germany, Hungary, Poland, Rumania, and the Soviet Union.

(ii) Each team consists of the eight students from the last two grades of secondary school who were winners in their own national olympiad.

(iii) On the first day, three problems are to be solved in four hours, and four problems in five hours on the second day.

(iv) The problems are very similar to those in the Hungarian competition, covering algebra, geometry and trigonometry, and requiring considerable ingenuity.

(v) In 1962, scores for the seven problems ranged from 5 points to 8 points each for a possible total of 46 points. First prizes were given to four students obtaining 41 to 46 points, second prizes to twelve students scoring 34 to 40, and third prizes to those fifteen students scoring 29 to 33. Two first prize students came from Russia, two from Hungary.

The Annual High School Mathematics Contest, jointly sponsored by the Mathematical Association of America and the Society of Actuaries, is the closest the United States comes to having a national contest. As of 1967, this contest had been administered eighteen times, (although nationally only since 1958). In1946, it was reported [27, p. 75] that the number of participants rose rapidly from 43,000 in 1959 to 200,000 in 1963. Some characteristics of the contest.

(i) Eighty minutes in length, covering arithmetic, two years of algebra, one year of plane genometry, and beginning coordinate geometry;

(ii) Questions are multiple choice, with a maximum possible score of 150.

The purpose of the contest is to contribute to the national search for brainpower, as is true of tests mentioned previously. (The director of one NSF-supported program claims that if a student does remarkably well on the MAA Contest, then he will have notable success in his summer program, with the MAA score being a better predictor than any other factor-test, grade average, background, etc.) Professor Salkind, contest director, has strongly urged that teachers use the test to supplement regular work rather than to stress competition with other students or schools.

An attempt is made to give everyone some success by intending that the first twenty questions be relatively easy. Were a student to get all twenty correct, he would score 60. Despite this, the median for only the <u>top</u> scorers in 1959 through 1963 were 32, 34, 31, 38, and 32, respectively. [A comment about the difficulty of the Mathematical Association test in relation to its format: like most of the other competitions described just below, it differs fundamentally from the Hungarian-type of contest in that it consists of many multiple-choice questions rather than a few problems requiring a completely written-out solution. Yet the MAA contest questions are considered quite difficult, and are as close in nature to the Hungarian-type problem as one could get via the multiple-choice format.]

Mathematical Association contest winners appear to be truly talented in mathematics. Out of fifty-nine test winners in 1958, Turner [32, pp. 425-6] found thirty-six had earned at least a bachelor's degree by 1963, with thirty-three in graduate school. Four graduate students reported that they would do either college teaching or research in mathematics. Of the twenty-three who had not yet received bachelor's degrees, fifteen were still undergraduates. One winner, a graduate student in mathematics at Berkeley in 1964, published an article in the Monthly in 1963.

Some universities have held contests on their campuses, apparently just as a matter of taking an interest in neighboring secondary schools.

At Sam Houston State College in Texas, a Saturday morning contest has been held for eight successive years. The test is in two parts, with each part consisting of multiple-choice questions written by members of the college's mathematics department. The test is scored in time for presentation of team and individual awards at a luncheon that same day. Each school team consists of two students who have completed at least two years of algebra and one of geometry. Each team is classified into one of two sections according to school population, with equivalent awards in each section. In 1966-67, thirty-five school teams competed.

It is unfortunate that time and space will not allow more than passing references to many other fine contests. (A survey by Howell Gruver of the Virginia State Board of Education -- to appear soon as a supplementary publication of NCTM -- has uncovered fifty-nine competitions in the United States [12, p. 1]). Some of these involve competition among schools from a geographic region:

(i) A statewide contest sponsored by the Tennessee Mathematics Teachers Association involves the cooperation of over twenty public and private Tennessee colleges. [The director of one Summer Science Training Program has noted that Tennessee contest winners tend to be remarkably fine mathematics students.]

(ii) The Tri-State Mathematics League (Maine, Massachusetts and New Hampshire) has nearly fifty schools competing in six interscholastic meets per year [23, pp. 38-40].

(iii) The interscholastic meets at Andrews High and Hockaday School, both of Texas, are sponsored by the respective Mu Alpha Theta Clubs.

(iv) The Alamo District Mathematics Contest of Texas had over 300 contestants from over 35 schools last year.

(v) Each of the eighteen schools participating in the Nassau County (New York) interscholastic league trains a squad of "Mathletes" to compete against other schools [18, pp. 113-14].

Some contests are intramural rather than interscholastic:

(i) At Sequoia Junior High [16, pp. 473-4], any student interested in mathematics can join a team that gives public demonstrations before school groups, civic organizations, and on local television.

(ii) In a Utah school [9, pp. 12-14], a field day includes five-minute talks by students, tests in computational skills and terminology, and mathematically-oriented games.

(iii) A St. Paul school has an <u>intramural</u> league among home rooms [35, pp. 386-7].

To summarize, contests have many beneficial effects. Hlavaty (33, p. 12] has noted that the interscholastic contests in New York City led to many students meeting after school on a regular basis to prepare themselves. Some teachers seized upon this chance to help students study problem-solving techniques and to lead them into independent study.

Some critics, on the other hand, question the highly competitive spirit that is generated through these contests, claiming that too much competition is not psychologically healthy. This can be particularly bad when all problems in a contest are completely beyond the capabilities of large numbers of entrants.

It would certainly appear that the European competitions enjoy a higher status among both professionals and the public than their American equivalents. Some claim that this difference results from the relative emphasis the two societies place upon intellectualism.

In any event, it could be easily documented that the various types of contests in this country have contributed to discovering unusual mathematical talent in secondary schools. Many teachers have devoted much hard work in seeing to it that contests exist and that their better students participate. Their efforts should be supported through more universities sponsoring programs like those at Stanford and Wisconsin. Such contests could lead to the recruitment of mathematically talented students and perhaps better school-college articulation. Colleges sponsoring summer study

opportunities like the NSF-supported Summer Science Training Programs might consider the merits of automatically admitting high scorers in the MAA contest or winners of worthy local or state competitions. Should one feel that contests can have bad psychological side effects, then the Wisconsin-style competition has merits in that the students solve their problems independently and voluntarily.

Group Study

There are many types of group study classifiable as extra-curricular activities for the mathematically gifted. Some examples:

(1) summer programs

(2) mathematics clubs

(3) seminars or lectures

The summer programs are predominant in number, financial support and quality. This is undoubtedly because they include the Summer Science Training Program (SSTP) supported by the National Science Foundation since 1958.

Among the purposes of the SSTP programs are the identification of mathematically gifted students and the subsequent attempt to give them insights into higher mathematics (science) and the work of mathematicians (scientists). For these reasons, the programs are usually administered by universities and held on their campuses. In 1967, there were 135 programs, of which 30 were devoted solely to mathematics while an additional 24 were part mathematics, part science. There were 1,542 participants in the mathematics-only programs, mostly rising seniors or juniors.

Although financed in large measure by NSF, each department of mathematics has had the freedom to devise its own program. Among the great variety of topics taught, the following have been chosen most often: some form of linear-algebra, algebraic structures, probability and statistics, computer

programing and computer-related mathematics, logic and set theory, number systems, number theory, foundations of mathematics, foundations of geometry. An attempt has been made to expose students to key mathematical ideas and techniques without giving them a complete course from the regular curriculum.

It would be impossible to report here on each program. It is almost equally difficult to select among them. But some have had the unusual feature of emphasizing independent projects, which qualifies them for special mention within the context of this report:

(i) In addition to courses in logic, modern algebra, probability and statistics, mathematics of sets, and principles of digital computers, the Rollins College program has had an independent study program that culminates in an oral report by selected students to the rest of the group [34, pp. 281-5].

(ii) According to Spira [30, pp. 87-9], the University of California at Berkeley program has tried to reproduce the atmosphere of productivity of professional mathematicians. Students are sought who have already started projects on their own. Difficult problems requiring original solutions are handed out on weekly problem sheets, with meetings held three times a week to discuss them.

(iii) The 1961 summer program at UCLA combined a formal course with a beginning research endeavor [2, pp. 276-8]. The introduction-to-research portion of the program consisted mainly of a library search by each student, following his selection of a topic interesting to him. The library search culminated in a paper. Among the topics: Game Theory; Godel's Theorems; Infinite Sets and Transfinite Cardinals; Introduction to Probability; The Mathematics of Voting. According to the author, all papers were interesting, and some were excellent, receiving favorable comment from members of the UCLA Mathematics Department.

(iv) Perhaps the most unusual independent study program has been offered for a number of years at Lehigh [38, pp. 250-4]. The intent here is to give students an experience equivalent to writing a thesis with the aid of a research mathematician. After several lectures, students work on simple projects. After several weeks of this, they attack "a major problem" with individual guidance. Some unusual results have been claimed.

Independent study is difficult to organize, requiring extraordinary amounts of instructor time of a special sort. What are its purposes? How

well does it work? Special information available about the Illinois programs of 1965, 1966, and 1967 and the Florida State programs of 1958-67 may help answer these questions.

In 1967 at Illinois, independent study was centered around the solving of difficult problems like those mentioned in the previous section on Contests. The 32 participants met twice a week to cover mathematical background essential to problem solutions. Then two groups of 16 each met for three 90 minute recitation sessions to dig deeper into individual problems and hear individual students report on proposed solutions. An instructor in that program reported by letter [36] that the purpose was to give high school students a chance to experience mathematical discovery. His claim was that deductively organized courses can sometimes stifle student thinking.

In 1965, the Illinois program was organized somewhat differently, with groups of five participants each selecting a topic to study under a faculty member's guidance. He reported [14] that group and individual results varied considerably. One problem: "Most of the groups waited until the last week to do their work and prepare their report. However, some of the students showed a real interest in such work, and one . . . expanded the ideas of his group in a paper for the Westinghouse Talent Search in which he placed fourth nationally."

Similar difficulties experienced in the Florida State University SSTP program have led to some special efforts. (Since the writer of this paper has taught for ten years in that program, special attention to it may be excused.) Since 1963, each participant has been required to do a directed individual study (DIS). Before arriving on campus, participants are prepared for this DIS activity through a questionnaire. During the first two weeks, each of five instructors lectures to a group of six students on his specialty. Participants are assigned to a group according to interests

stated in the questionnaire. For the remaining four weeks of the six-week session, students are pretty much on their own, except:

(i) each student meets his directed individual study instructor at least once a week for an oral report and further guidance;

(ii) each student submits a written report by the end of the fifth week;

(iii) each student orally reports to other students during the sixth week.

Each DIS instructor takes whatever additional steps are necessary to keep each student moving.

This schedule is the result of DIS instructors being uncomfortable with student effort and accomplishment in the first year. These instructors have claimed that students experience two difficulties in carrying out an independent study:

(i) it is hard to select a topic;

(ii) once selected, it is hard to "keep one's nose to the grindstone."

Both difficulties seem to stem from one or more of several causes:

(i) high school students have usually had no experience in independent study; it is seldom emphasized by teachers below the college level;

(ii) even the mathematically bright high school student has, relatively speaking, little mathematical knowledge -- at least not always enough on which to base independent study. Despite such excellent monographs as those in the SMSG New Mathematical Library, there are few materials at an appropriate level, or not all school libraries contain them;

(iii) time is at a premium even for high school students; the conscientious student often feels that daily requirements have to come ahead of any independent work.

For a student to overcome these difficulties requires a great deal of motivation and interest on his own part, and considerable wisdom, knowledge, and expert guidance on the part of his instructor. Perhaps more needs to be done to improve the instructor's competence in this regard. On the other

hand, instigating and completing an independent study may be a real test of a student's mathematical competence -- i.e., a kind of selection technique. It has been so regarded by instructors at the FSU summer program.

Many other SSTP programs have had unusual features of special interest. Arnold Ross, first at Notre Dame and then at Ohio State, constructed a problem assignment to help select participants [26, p. 440-43]. Some unusually talented students have been invited back for a second year; a follow-up during the academic year is now being worked on.

Case Institute has a six-week summer program, but the top one-third of the participants are invited to stay on for an additional five weeks. Stevens Tech has a Saturday morning program that covers the same subject matter as in the summer. Grossman [12, p. 75] reported that the Columbia program was an outgrowth of a Science Honors Program in New York City, held on Saturday mornings since 1958.

For the reader who wishes to look into other excellent SSTP programs, some of the many descriptions appearing in the literature have been listed in the References. Included among these are reports of student evaluations, which have generally been quite complimentary [21, pp. 149-54]. (Ettlinger of the University of Texas has reported by letter that he now is putting together a follow-up of participants in past Texas programs.)

Some local communities and states have instituted group-study programs of their own:

(i) Herbert Ware, Supervisor of Mathematics for the Arlington County Public Schools of Virginia, has reported (by letter) that his school system offered in the summer of 1967 a special course entitled Logic: A Games Approach. Based on the Wif'N Proof kit, the course lasted for sixteen days for five hours a day. Any 8th - 12th grader could enroll. Two other courses, Mathematics through Science (SMSG) and Enrichment Mathematics (independent study) were planned, but then cancelled due to insufficient enrollment. Similar offerings are planned for 1968.

(ii) A more formal program is offered at St. Paul's School of Concord, New Hampshire. Called the Advanced Studies Program, it has several purposes: providing gifted students with otherwise unavailable opportunities; interesting prospective teachers in teaching; and providing inservice teachers with instruction in teaching the talented. Two mathematics courses are offered: Topics in Mathematical Analysis (study of the limit concept), and Concepts of Mathematics (topics from set theory, logic, algebra, number theory, and probability).

(iii) Cooperation between school system and local industry led to a computer programing class in Chula Vista, California in 1962 [37, pp. 340-3]. There were two hours of instruction per day for six weeks, supplemented by use of the UNIVAC 80 at Rohr Corporation. Students were twenty-one rising sophomores from six junior highs. Eleven students had had one year of algebra, the others had had none.

(iv) Similar cooperation has led to an annual summer program for thirty bright eleventh-graders at the Thacher School of Ojai, California. Combined with offerings in the natural sciences, the program touches on mathematical topics from spherical trigonometry, analytical geometry, basic calculus, elementary differential equations, vector analysis, and programing of the CD G-15 and CDC 3600 digital computers.

(v) According to a letter from Supervisor of Mathematics Loetta Horton, the Roanoke City Public Schools have cooperated with the county school system, the Virginia Society of Professional Engineers, and various industries to offer a combined science and mathematics summer program for talented students who completed two years of algebra. The mathematics consists of a short course in matrix algebra. Similar cooperation has also led to a series of engineering lectures during the academic year.

(vi) A letter from Robert Jones, North Carolina State Supervisor of Mathematics, revealed a novel seven-week statewide summer school called the Governor's School of North Carolina. Four hundred talented high-school juniors and seniors have attended in each of the past five years. Of this number, about fifty study mathematics. Apparently, attendence is by invitation only.

Another type of group-study extra-curricula activity is the mathematics club. The few articles on mathematics-club activities [see 11, 20, and 31] indicate that at a typical club meeting students solve puzzles, attempt complex problems, listen to lectures on mathematics or mathematical careers, plan contests (see previous section), or engage in some kind of

social activity. One author [31, pp. 715-18] reported an emphasis in his school's club upon each member's undertaking an independent project, then reporting his results to other students. This requires, the author claims, a membership restricted to students of superior ability who seem likely to become mathematicians and research scientists. A letter from the State of Oregon mathematics supervisor reported that the Kingsmen Math Club at Rex Putnam High sponsored a very unusual one-day mathematics conference at which mathematicians from nearby colleges gave lectures appropriate to high school students.

Such a wide variety of programs is apparently also true in the school mathematics clubs of the U.S.S.R., according to Gnedenko [10, pp. 384-403]. He does note a tendency to observe anniversaries of prominent mathematicians (these need not be Russian!) by undertaking historical reports, and refers to special pamphlets that have been published for these clubs. (These include some translated into English and published by MIT Press.)

Gnedenko does note a type of math club that apparently has no analog in the United States -- mathematical circles attached to Universities. Here, undergraduate and graduate students are in charge of teaching the school students, usually by presenting problems to be solved. These circles are related to the previously described Russian Olympiads (contests).

Comment: the writer is left with the impression that the mathematics clubs of the United States are, on the average, weakly organized, with few programs of interest to the students, and having a small membership. A national math club honorary, Mu Alpha Theta, does exist with chapters in quite a few high schools. Despite leadership provided by some faculty members at the University of Oklahoma, individual chapters appear to be left pretty much on their own in the way of program development. Sustained leadership and financial support, similar to that given by universities and

NSF to summer programs, are vitally needed. The Russians have set an example in this regard.

Some support has been provided in the form of occasional lectures by outstanding mathematicians. For a while the Mathematical Association of America, with NSF support, administered a visiting lecturer program in local schools. The American Association for the Advancement of Science currently sponsors a Holiday Science Lecture Series, again with NSF support. This year, about eleven different lecturers will visit as many cities for about two days during the Thanksgiving or Christmas holiday seasons. Only one mathematician, Mark Kac, is on the AAAS list of lecturers, however.

Independent Study

Aside from its occurrence in some SSTP programs, the technique of independent study by mathematically gifted secondary school students has been promoted through science fairs and other enrichment programs.

Of the former, a key program is the Westinghouse Science Talent Search [5]. Sponsored by the Science Clubs of America and administered by Science Service, this annual program is now in its twenty-seventh (27th) year. Among its characteristics:

(i) Only seniors from a U. S. secondary school are eligible.

(ii) An entry consists of a score on a nationally administered science aptitude test, a completed personal data blank, a secondary school transcript, and a 1000-word report on an independent research in science or mathematics.

Awards are given as follows:

(i) The forty contestants judged best by a committee designated by Science Service are given all-expenses-paid trips to a Science Talent Institute, (to be held in 1968 at Washington, D.C. from February 28 through March 4.)

(ii) Ten of these forty Institute participants will be given four-year scholarships, ranging from a first prize of $2,500 per year to four fourth-prizes of $1,000 per year each. The scholarship may be applied toward a course in science or engineering at a degree-granting institution of higher education selected by the winner and approved by a Science-Service-appointed scholarship committee. Science and Engineering courses are defined as any encompassed within the fields of the National Academy of Sciences, the National Research Council, and the National Academy of Engineering.

The titles of winners' reports for a previous year are impressive. There is but a minor fraction devoted to mathematics, however. Science Service claims that the records of the 1,040 past winners (40 per year for 26 years) indicate that this Talent Search has accomplished its purpose:

(i) All participants have attended college. Of those having the opportunity thus far, almost all have obtained bachelor's degrees, and ninety percent of these have gone on to a Ph.D., M.D., or equivalent degree.

(ii) Most having completed their formal education have become college professors, with the next largest group employed as research scientists in industry.

(iii) Ninety-nine percent have chosen a branch of science as a career, with mathematics, physics, and chemistry being their first choices.

Related to the Westinghouse Science Talent Search is the program called the International Science Fair [5]. A science fair is an exhibit of the results of independent study projects in natural sciences or mathematics. Also administered by Science Service, this program for secondary school students is carried out annually as follows:

(i) Almost any science teacher, with appropriate acknowledgement by local authorities and help from Science Service, may organize a fair at the local (school, school district, or city) level.

(ii) Those student projects judged best at local fairs are brought together in a regional (state or other geographic entity) fair. There are about 220 regional fairs each year.

(iii) Students in the 10th through 12th grades exhibiting projects judged best in the regions are invited to enter their exhibits at an International Fair.

Among countries participating in the international program, held annually since 1950, have been Canada, Germany, Japan, Nicaragua, Philippines, Portugal, Sweden, Switzerland, Turkey, and the United States. (National science fairs have been held in many other countries as well.)

Comment: again, as one might expect, there seems to be a preponderance of <u>non</u>-mathematical projects exhibited in these fairs. It seems unfortunate that the mathematically bright have to turn to a science-dominated program to find an outlet for the results of their independent studies. Some thought should be given by such organizations as NCTM, MAA, and Mu Alpha Theta to the sponsorship of a <u>Mathematics</u> Talent Search and/or a <u>Mathematics</u> Fair of an appropriate sort.

Outside the realm of such externally organized programs as the Westinghouse Talent Search and the International Science Fair, mathematics

teachers have devised techniques of their own within their classrooms. These are often called "enrichment programs." One mathematics educator surveyed ten other prominent colleagues as to their suggestions for the mathematically gifted [17, pp. 322-25]. In addition to techniques already discussed were the following:

(i) a suitable library of texts and periodicals to appease curiosity and to develop informal reading interests;

(ii) correspondence courses;

(iii) statistical surveys in the community;

(iv) preparation of models.

Comment: the suggestion about correspondence courses seems particularly interesting. Since correspondence courses are normally offered to adults and college students, a special set of courses would have to be developed for mathematically talented secondary school students. This has been done in Russia, where the Soviet mathematician I. M. Gelfand organized in 1963 a Correspondence School at Moscow State University. The MIT Press catalog for 1967, in describing its American editions of the Library of School Mathematics series that were prepared by the Survey of Recent European Mathematical Literature at the University of Chicago, mentions that 1,000 ninth grade pupils were admitted as correspondence students in 1966. The writer of this paper could not uncover the existence of any equivalent activity in the United States. Something should be done about this.

It has been recommended that each teacher collect sets of challenging problems for use with his own classes [15, p. 79]. The problem can be placed on 3 by 5 cards, then filed. Bright students should be encouraged to check out the cards and attempt solutions on their own. The teacher helps with hints as necessary. Solutions can be reported orally to the rest of the class.

Independent study projects can be undertaken within the context of a

regular school program. At Phillips Andover Academy [19], any senior can do one in place of a required course or of one term of athletics, if a faculty member agrees to serve as sponsor. One instructor sponsored a student's reading in probability of part of Feller's text and of Cramer's Elements. (This instructor, however, expressed some disappointment in his evaluation -- "I cannot honestly say that accomplished too much.")

A teacher has reported some Nebraska schools using these techniques [22, pp. 339-45]:

(i) Teachers encourage pupils to do some minor library research that leads to publication in a special school newspaper entitled Math News.

(ii) Group as well as individual projects are encouraged. These are then exhibited at a Math Fair in Omaha.

(iii) Assembly programs in mathematics are given. One guest lecturer spoke on careers in mathematics, enhancing his lecture with student participation.

This author claimed that "many . . . resource units for the teaching of the gifted mathematics pupil have been developed and are readily available to any school." Comment: the writer of this paper has not seen any resource units of this sort. The idea is an excellent one; much work should be done to develop many such resource units and to make them generally available.

Also, the suggestion of the Math News recalls to mind the NCTM Mathematics Student Journal. It has not been brought to the attention of mathematically talented students throughout the country as much as it should. (An encouraging notice in a recent NCTM Bulletin for Leaders indicates that an effort along these lines may soon be undertaken.)

A further comment: the library-search suggestion raises the question as to whether school libraries are stocked with a sufficient number of appropriate materials. Mu Alpha Theta and the National Council of Teachers of Mathematics have published book lists. But there seem to be only a few

outstanding schools that have an active and regular book, journal, and monograph acquisition program.

There is also a serious question as to whether there is a sufficient variety of materials pitched at a level appropriate to senior high, let alone junior high students. The monographs in the SMSG New Mathematical Library and the D. C. Heath Russian translations are excellent; more should be published, then made known to more students. But easier, shorter publications are also urgently needed.

As usual, one should not forget the importance of the teacher, even vis-à-vis the use of a library. Some SSTP students claimed their greatest summer's pleasure was the opportunity to browse through an excellent library. But even that joy changed to confusion whenever guidance was not given in selecting texts or in explaining difficult passages.

The work of some mathematical educators suggests that some of the new media could be used to give mathematically talented students an increased opportunity to study special mathematical topics on an extra-curricular basis. Berger [3] claimed success in teaching enrichment topics by means of television to ninth and tenth graders. Dessart [8, p. 1499] used programed texts to teach some ideas about convergence of infinite sequences to eighth graders.

The computer holds many possibilities for the mathematically bright. Pieters reports in a letter: "The Time-Sharing connection we now have to the Dartmouth Computer has led to some very interesting individual projects." Robert Jones, State Supervisor of Mathematics Education in North Carolina, has reported in a letter that a time-sharing computer project is being conducted at Needham Broughton High School in Raleigh for mathematically gifted students. Many mathematically talented students have become fastinated with programing the computer while attending an SSTP program

[12]. Some have gone on to secure part-time employment as computer programers, or have pursued independent projects using the computer. The Association for Educational Data Systems has given prizes [1, p. 11] for unusual computing efforts.

Teaching

Time and again, mathematically talented students in SSTP programs have complained of boredom during their regular mathematics classes in their home schools. They remark that the only bright spots occur when they are allowed to progress on their own, or when they are asked to tutor weaker students.

One finds occasional side references in the literature to the idea of having mathematically gifted students serve as tutors or teachers. But there is no indication that anyone has explored this technique in any detail. What are its possibilities? Should gifted students be paid as teacher aides? Would there be enough intellectual return to the gifted student for this investment in his time?

Another related technique is the symposium. For example, 300 mathematically and scientifically talented secondary school students in the metropolitan New York area met at the IBM-Junior Science Symposium in October of 1960 [28, pp. 293-4]. Edward Teller spoke on "Geometry of Space and Time." Then six high school students with unusual records gave talks on their own research. These included two mathematical reports:

(i) Matrices and determinants, by James Pepe of Xaverian High;

(ii) The use of the digital computer in the investigation of Fibonacci numbers, by Harry Saal of Midwood High School.

A letter from the Oregon mathematics consultant contains a description of an annual mathematics conference sponsored by the mathematics club of Wilson High School in Portland. The high school students in the club

compete for the privilege of presenting papers on mathematical topics to junior high school students in their attendance area.

Comment: the idea of a conference or a symposium seems to have special merit. Either one seems to fit mathematics better than a fair. They can overcome the disadvantage of a contest cited by Radó, (i.e., some bright students can do little with a problem in a timed testing situation). Even in small communities enough talented students could be found by including a larger geographic region; there would be merit in holding such meetings on a university campus. Much more needs to be done to give mathematically talented students opportunities like these to present the results of their independent efforts to their peers. Pi Mu Epsilon chapters might very well consider the merits of sponsoring such affairs.

In conclusion: it can be seen that there are many extra-curricular activities available in the United States for mathematically talented secondary school students. Several are of excellent quality, and imaginative in design or execution. But, with the exception of the SSTP programs and the MAA Contest, few have been sufficiently well organized and promoted with the real urgency these gifted students deserve.

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Some of the Organizational and
Administrative Problems in Schools
With Programs for Gifted Students

Marie Wilcox

Talking about what can be done for the gifted is one thing, actually putting plans into action is another. In some of the large cities separate schools are provided for the students. This is true in New York and Cincinnati for instance. Chicago does not have schools designated as such but students may attend the school of their choice, and I understand that the gifted just naturally drift toward certain schools. Chicago also has special classes in other schools and provides all schools with Curriculum Guides for the honors and advanced placement classes.

Some school systems rely only on separate classes within the school to care for the talented students. For example, the system in Jefferson County, Kentucky (around Louisville) does this. They have special classes from grades 2 through 12. If a student who should be in one of these classes is in a school too small to schedule such a class, the student is transported to another school. This school system also provides guides for the courses and has one supervisor whose entire duties are to assist with the organization and operation of these classes.

Even in small schools special classes are usually organized. Dr. Conant, in his report to NASSP earlier this year, stated that replies from questionnaires indicated 96.5 percent of those replying had some ability grouping. Where there is no grouping according to ability in mathematics, I assume that it is left for the classroom teacher to use differentiated assignments to challenge the gifted.

Within a school problems arise in the organization and teaching of these

special classes.

1. Assignment of students to the classes. This is usually done by test results and teachers' recommendations. Then the student is informed that he has been recommended and the assignment is described as an honor and an opportunity. However, if he or his parents request that he not be placed in the class, it is not done. The majority of the students accept and many parents are glad for the opportunity which the child will have. On the other hand, some parents complain when their children are not invited into the special classes.

Some of the reasons why a student does not accept the assignment to honors classes.

a. Fear that his grade average will be lower than it would be in regular classes and he will not be admitted to the college of his choice. In some cases, he or his parents are just concerned about his standing in the senior class.

b. Concern that the assignments in the class will be too lengthy and he will not have enough time for participation in other activities, such as athletics, musical events, and a job outside of school.

c. Dislike for the particular teacher assigned to teach the course.

d. Concern that he may be assigned to too many honors sections and that he cannot do justice to all. Parents often interfere here and suggest a limited number of honors sections.

2. Organization of classes. In some schools there are not enough students who qualify for an honors class. School systems frequently will not support small classes. Possible handlings of the situation are: (a) transportating these students to another school, (b) enlarging the class to an acceptable size by including students not as capable, and (c) having no honors class.

In a school it might be that there are 38 who are eligible for an

honors class. This is probably too many for one class and too few for two classes. The usual solution is to omit some of the students. The parents of these students would usually say that they consider this a serious problem.

3. Assignment of teachers for honors classes. In general there are two problems.

a) Some qualified teachers prefer not to teach these classes.

b) Some teachers not qualified to teach the courses ask why they do not get such assignments. This creates a personnel problem within a department.

4. Qualifications of teachers. Qualifications for a teacher for such a class at the secondary school level are probably something like this.

a) The teacher should have at least an M.A.T. degree in Mathematics from a university where the department of mathematics is known to give a fine program. He must keep up to date in subject matter.

b) On the other hand, the teacher should not be a person who wants to have the center of the stage and prefers to show the class how much he knows instead of letting the students show how much they know.

c) The teacher should be resourceful and versatile -- he must be the kind of person who can change the entire plan for a lesson when something happens which makes this seem best for the class. The change might be made in the middle of a recitation period or in the plans for future lessons.

d) He must not be the kind of person who makes an effort to catch the student in petty errors or is prissy about the method by which problems are solved.

e) He must be liked and respected by the students. Usually this

means that he maintains an atmosphere in the class which will induce a learning situation. He is impartial in his dealings with the students, fair in grading, and recognizes what constitutes a reasonable assignment.

f) He must be willing to say that he doesn't know the answer to a question and to find the answer by reference to books or people from whom he can get the answer.

g) He must want to teach these students.

Problems may arise when the administrator concerned thinks that the students are assigned correctly and the best choice of teachers has been made. If lesson assignments are unreasonable or the grading too low for this caliber of students, students will ask to be transferred out of the honors classes. A careful check of the grades in honors classes probably should be made by the department chairman or school principal and discussed with the teacher if necessary. In my particular school usually the teachers explain to me, without my asking, anything unusual. In the department we have midterm and final examinations. The grading scale is made by considering all the grades in the department at that level. If all the students in G-classes are in the A range, they all receive grades of A. As for assignments, it is generally agreed, I believe, that the quality and not the quantity of the work should be different. The student should have a few challenging problems and not just twice as many exercises as a regular class.

5. Problems which occur within the classroom.

a) The most serious problem is that of motivating the extremely capable student who will not study. Some of these students are satisfied to learn that they can learn by listening in class, and the teacher finds it difficult to persuade the student to do independent study.

b) There are a few cases when the student is not prepared well enough

to pick up the pace in the class. If he is capable, eager and willing, he usually can correct this difficulty with additional study and some teacher guidance. If not, he probably needs to be assigned to another section. This happens at times when a student transfers from one school to another one. An honors class in one school is not always the same as that in another school.

c) If a student has attended a NSF high school institute, he sometimes has had so much of the material in part of a course back in his own school that he becomes bored. If his high school has nothing more challenging to offer, it is sometimes recommended that he enroll in a college course (if a college is available) or in a correspondence course from a college.

d) Some students complete elementary calculus in their junior year in high school. We have quite a number in the greater Indianapolis area. They usually enroll in second year calculus at the Purdue or Indiana University Extension divisions in Indianapolis during their senior year.

e) I spoke to a very intelligent parent of a gifted student in Texas last weekend, and she says the students in honors sections complain about the volume of work (not in mathematics but in social studies). They also complain about the intermediate algebra class which is not an honors class and moves too slowly. The students feel that courses beyond intermediate algebra pose no problems since there is a sort of natural selection of better students.

6. Extra-curriculum activities. Often students extremely gifted in mathematics are not interested in mathematics clubs which have meetings after school. Many of the students have other after-

school activities such as debate practice, athletics and rehearsals for musical or dramatic productions. Others have a transportation problem if they remain after school hours.

Such students also may feel they do not have time for extra reports, extensive outside reading or participation in contests which require a considerable amount of review.

Recommendations of the Conference

I SMSG should give highest priority to the development of supplementary materials for gifted students. While there is a need for courses explicitly designed for gifted students, on the one hand, such courses are being developed by other organizations, and, on the other hand, many schools are unable to provide classes exclusively for gifted students. Supplementary materials for gifted students in classes of primarily average and above average students are therefore highly desirable. Such supplementary materials should be articulated with a standard course for above average students and should concentrate on greater depth and on applications of the topics covered in the course. (For example: To accompany a unit on multiplication of terminating decimals, additional materials could be made available which would encourage the gifted student to investigate multiplication of non-terminating decimals, or, in another direction, the effects of round-off error.) Some of these materials might be purely expository but many of them should be open ended to provide the gifted student with an opportunity for creative work. In any case, a wide variety of such materials is needed.

II For extracurricular use, a wide variety of topics for investigation and open ended research problems should be available to gifted students. Presently extant materials should be reviewed and a bibliography of them should be prepared. In order to provide the wide variety which should be available additional units of this kind should be prepared.

III For the benefit of gifted students in schools where the local staff, for one reason or another, cannot provide appropriate guidance,

correspondence courses should be developed. These should be short, possibly one semester in length. For each a structured outline should be prepared and a number of alternative texts suggested. Extended and frequent student-instructor interaction is recommended to minimize dropouts..

IV In order to make the materials recommended above effective, teachers will need to be provided with appropriate materials and guidance. It is also recommended that NSF support summer and in-service institutes to train teachers who will be working with gifted students.

V A wide variety of short expository booklets should be available to gifted students. A list of presently available materials of this kind should be prepared. The SMSG Supplementary and Enrichment series should be extended. In particular, much more material for junior high school students is needed.

VI Local and regional symposia for junior high school gifted students should be encouraged.

VII Mathematicians from colleges, universities and industry should be encouraged to provide assistance and guidance to gifted students in nearby schools. (This would be an extension of the SMSG program for extraordinarily gifted students.)

VIII Local, regional, and national mathematics competitions should be encouraged. The East European Olympiads should furnish some useful suggestions.

IX It is recommended that SMSG attempt to identify student characteristics, other than IQ, which characterize mathematically gifted students.

X It is recommended that SMSG and CUPM appoint a joint committee to study the problems of the transition of gifted students from school

to university.

XI Although most of the above refer to the secondary school level, gifted students in the elementary school deserve attention. It is recommended that SMSG continue to investigate this problem.